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Nucleation of Quark–Gluon Plasma Droplets

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Abstract

The energy densities achieved during central collisions of large nuclei at the AGS may be high enough to allow the formation of quark–gluon plasma. We propose that most collisions at AGS energies produce superheated hadronic matter, but in rare events a droplet of quark–gluon plasma is nucleated. We estimate the probability of this to occur based on homogeneous and inhomogeneous nucleation theory and discuss possible experimental signals.

I. INTRODUCTION

One of the mysteries of heavy ion physics at Brookhaven National Laboratory's AGS is: *If hadronic cascade event simulators like RQMD , ARC and ART [1] produce energy densities approaching 2 GeV/fm^3, yet agree with experiment, where is the quark–gluon plasma?* After all, numerous estimates of the onset of quark–gluon plasma agree that it should occur at about that energy density, and if there is a first order phase transition, then the onset of the mixed phase would occur at an even lower density.

A possible answer is, that *most collisions at AGS energies produce superheated hadronic matter and are describable with hadronic cascade simulators, but in rare events a droplet of quark–gluon plasma is nucleated which converts most of the matter to plasma.*

We investigate two possible production mechanisms: homogeneous [3] and inhomogeneous nucleation theory [4]. In the first mechanism we estimate the probability that thermal fluctuations in a homogeneous superheated hadronic gas could produce a plasma droplet, and that this droplet was large enough to overcome its surface free energy to grow. In the second mechanism we consider another source of plasma production which is essentially one of nonthermal origin. We estimate the probability that a collision occurs between two highly energetic incoming nucleons, one from the projectile and one from the target, that this collision would have produced many pions if it had occurred in vacuum, but because it occurs in the hot and dense medium its collision products are quark and gluon fields which make a small droplet of plasma. All our calculations apply to the reaction Au+Au at 11.6 GeV/A.

The production of a plasma droplet should have experimental ramifications. Since the phase transition to the quark–gluon plasma is occurring so far out of equilibrium we would expect a significant increase in the entropy of the final state. This could be seen in the ratio of pions to baryons or in the ratio of deuterons to protons. Along with the increased entropy should come a slowing down of the radial expansion due to a softening in the matter, that is, a reduction in pressure for the same energy density. Together, these would imply a larger source size and a longer lifetime as seen by hadron interferometry. Another dramatic effect is the development of a shoulder in the charged particle multiplicity distribution. The shoulder becomes more pronounced the more likely quark–gluon plasma production becomes and therefore the more charged mesons we produce in this new class of events.

II. HOMOGENEOUS NUCLEATION THEORY

We need three ingredients to estimate the probability for quark–gluon plasma production using homogeneous nucleation theory: an equation of state, the nucleation rate for plasma droplets and a global description of the dynamical evolution of the nuclear collision.

The dynamics of a central nucleus-nucleus collision at the AGS is extremely complicated. We shall be satisfied here with a simple model. Imagine the colliding nuclei as two Lorentz contracted disks in the center of momentum frame. At time $t = 0$ they touch. They interpenetrate between $0 \le t \le t_0$ where $t_0 = R/v\gamma$, R and v are the nuclear radius and velocity, and γ is the Lorentz factor in the center of momentum frame. At the end of this time the nuclei are completely stopped. The volume of overlap is a linear function of time.

After the time t_0 the hot fireball expands radially. At late times we would expect its radius to grow linearly with time. Therefore we parametrize the volume as $V(t) \sim (t + c)^3$. The

constant c and the proportionality factors are determined by matching the two functional forms for the volume and their first derivatives at t_0.

To determine the equation of state for baryon rich matter we assume a first order phase transition. The quark-gluon plasma phase is described by a free gas of massless quarks (u, d, s) and gluons confined by a bag constant B = (220 MeV)^4, while the hadronic phase consists of a gas of mesons (π, K, K^*, η, η', ρ, ω, ϕ and a_1) and baryons (N, Δ, Λ and Σ) interacting via a repulsive mean field of strength K = 1500 MeVfm^3. Both phases are joined through a Maxwell construction.

Given the lab kinetic energy of the collision we can determine the initial energy density and baryon density in the overlap region and the starting point of the superheated hadronic system in the $T-\mu_B$ plane. Assuming baryon number conservation and an entropy conserving hydrodynamic expansion we determine the time evolution of the system in the $T-\mu_B$ plane. The system reaches the coexistence curve after a time $t_f = 7$ fm.

The rate I to nucleate droplets of quark–gluon plasma in a hadronic gas per unit volume per unit time is given by

$$I = I(\mu_B, T) = I_0 \ \exp(-\Delta F_*/T)\,. \tag{1}$$

Here I_0 is the prefactor and ΔF_* is the change in free energy of the system due to the formation of a single critical size droplet of plasma.

The nucleation process is driven by statistical fluctuations which produce droplets of quark–gluon plasma in the hadronic phase. The size of these fluctuations is determined by the free energy difference of the hadronic phase with and without the plasma droplet. The system under discussion is in a superheated state so that the interplay between the pressure difference and the surface free energy results in a maximum of the free energy difference at the critical radius $R_*(T, \mu_B)$. Droplets with a radius larger than R_* will expand into the hadronic phase, while droplets with a radius smaller than R_* will collapse. ΔF_* is the activation energy needed to create a droplet of critical size R_*.

The prefactor I_0 is linearly proportional to the dissipative coefficients, as expected for linear viscous hydrodynamics. For the droplets to grow beyond the critical radius, latent heat must be carried to the surface of the droplet. This is achieved through thermal conduction and/or viscous damping.

Nucleation begins when the two nuclei first collide with each other and a superheated overlap region is created. It ends when the expanding system reaches the phase coexistence curve at time t_f. The average droplet density can be computed from the expression

$$n_{drop}(t) = \int_0^t dt' I(\mu_B(t'), T(t'))\,. \tag{2}$$

The maximum possible value reached by the droplet density at t_f is $n_{\rm drop} = 2 \times 10^{-5}$ fm^{-3}. The volume of the expanding system, on the other hand, is on the order of 2×10^3 fm^3. The average number of droplets nucleated is thus rather small, roughly 1/100 for a central Au + Au collision at the AGS. We found furthermore, that the volume fraction q of quark-gluon plasma is less or on the order of 10^{-2}. Finally, we calculated the average radius of a droplet

produced $\bar{R}$ and found the average radius to be less or on the order of 5.0 fm at the time t_f when the system hits the coexistence curve.

We see that in an average central collision at AGS we produce hardly any quark-gluon plasma (q is very small) and hardly any plasma droplets are created ($n_{\rm drop}$ is very small). But, if a droplet should be produced it will grow and fill a sizable part of the system ($\bar{R}$ is large). We conclude that perhaps in one out of every 100 or 1000 central collisions a sizable fraction of the system has undergone the phase transition into quark-gluon plasma.

III. INHOMOGENEOUS NUCLEATION THEORY

In inhomogeneous nucleation theory we study the initial stage of the heavy ion collision. A small but growing overlap region consisting of stopped, hot and dense hadronic matter has already formed, but additional target and projectile matter is still colliding with this hot and dense zone. We are specifically interested in the possibility that an incoming projectile nucleon suffers little or no energy loss during its passage through the hot and dense zone where it encounters a target nucleon which also has suffered little or no energy loss. The energy available in the ensuing nucleon-nucleon collision, $\sqrt{s}$, can go into meson production. Suppose that a large number of pions would be produced if the collision had happened in free space. Clearly, the outgoing quark and gluon fields cannot be represented as asymptotic pion and nucleon states immediately. The fields must expand and become dilute enough to be called real hadrons. If this collision occurs in a high energy density medium, the outgoing quark-gluon fields will encounter other hadrons before they can hadronize. It is reasonable to suppose that this "star burst" will actually be a seed for quark-gluon plasma formation if the surrounding matter is superheated hadronic matter.

A fundamental result from kinetic theory is that the number of scattering processes of the type $1 + 2 \rightarrow X$ is given by

$$N_{1+2\rightarrow X} = \int dt \int d^3x \int \frac{d^3p_1}{(2\pi)^3}\, f_1(\mathbf{x}, \mathbf{p}_1, t) \int \frac{d^3p_2}{(2\pi)^3}\, f_2(\mathbf{x}, \mathbf{p}_2, t)\, v_{12}\, \sigma_{1+2\rightarrow X}(s_{12})\,. \tag{3}$$

Here v_{12} is a relative velocity and the f_i are phase space densities. A differential distribution in the variable Y is obtained by replacing the interaction crossection σ with $d\sigma/dY$.

For our purpose it is reasonable to represent the colliding nuclei as cylinders with radius R and thickness L. All the action is along the beam axis. We assume that the phase space distributions are independent of transverse coordinates x and y and of transverse momentum. The phase space density of nucleon i can then be rewritten as

$$\frac{dp_{zi}}{2\,\pi}\, f_i(z, p_{iz}, t) = \gamma\, n_0\, \frac{dx_i}{x_i} \sum_{N_i=0}^{\infty} H(x_i, N_i)\, S(N_i, d_i(z,t))\,, \tag{4}$$

where n_0 is the average baryon density in a nucleus, about 0.145 nucleons/fm^3. Here, $H(x, N)$ is the probability that the nucleon has momentum fraction x after making N collisions and $S(N, d)$ is probability that the nucleon has made N collisions after penetrating to a depth

d. $\sum_{N=0}^{\infty} H(x,N)S(N,d)$ is then the probability that the nucleon has momentum fraction x after penetrating to a depth d.

For the survival function $S(N,d)$ we assume that the collisions suffered by the nucleons are independent and can be described by a Poisson distribution characterized by the mean free path $\lambda \sim 0.4fm$ of nucleons in the hot and dense hadronic matter. The invariant distribution function $H(x,N)$ describes the momentum degradation of a nucleon propagating through the hot zone. Csernai and Kapusta [5] evaluated H under the assumption that the nucleons experience deceleration by sequential scattering. Their result depends on one parameter w, the probability that the nucleon collides inelastically. Using $w \sim 0.5$ allowed them to obtain a good representation of p+A and n+A data with beam energies in the range of 6-405 GeV.

The spectrum dN/ds is plotted in the Figure to the left. The nucleon-nucleon collisions may be referred to as primary-primary, primary-secondary, and secondary-secondary, depending on whether the nucleons have scattered from thermalized particles in the hot zone (secondary) or not (primary).

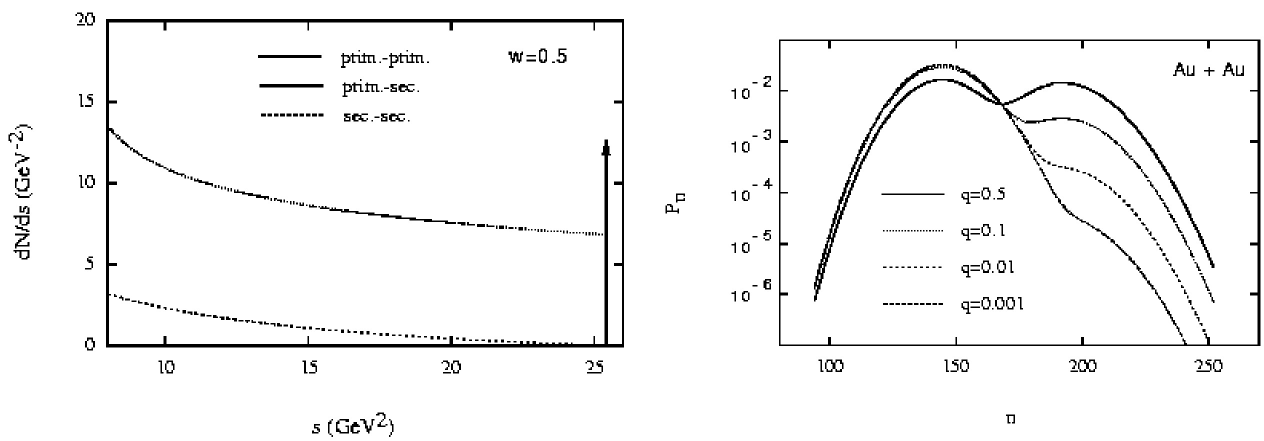

FIG. 1. Left: Distribution dN/ds of hard nucleon–nucleon collisions taking place in the hot zone. Right: Negatively charged multiplicity distribution for different values of q

Until now we have only selected nucleon-nucleon scatterings in which the total available energy $\sqrt{s}$ is large. In addition, we need to specify what fraction of this energy goes into meson production. This is described by the pion number distribution function $P_n(s)$, which is the probability of producing n pions in a nucleon–nucleon collision in free space. Given $P_n(s)$ we can estimate the number of nucleon-nucleon collisions that would lead to the production of n pions as

$$N_n = \int_{s_{\min}}^{4E_0^2} ds\, P_n(s)\, \frac{dN}{ds}(s)\,. \tag{5}$$

The lower limit of integration is fixed by kinematics and the upper limit by the beam energy.

We shall approximate the pion number distribution function $P_n(s)$ with a binomial.

$$P_n(s) = \begin{pmatrix} n_{\max} \\ n \end{pmatrix} \xi^n (1-\xi)^{n_{\max}-n} \tag{6}$$

The parameter ξ is related to the mean multiplicity by

$$\xi(s) = \frac{\langle n(\sqrt{s})\rangle}{n_{\max}(\sqrt{s})} \tag{7}$$

Here $\langle n\rangle$ is the average pion multiplicity averaged over pp, pn and nn collisions and can be taken from experimental data. The maximum number of pions produced in a nucleon-nucleon collision $n_{\max}$ is determined by kinematics. N_n allows us to evaluate the number of hard nucleon–nucleon collisions, producing n pions in free space. This quantity is our measure for seeds or nucleation sites of quark–gluon plasma production.

We are interested in the possibility that one of these seeds nucleates plasma. The precise criterion for this to happen is not known. However, we can make some reasonable estimates. In [3] we estimated that a critical size plasma droplet at these temperatures and baryon densities would have a mass of about 4 GeV. Any local fluctuation more massive than this would grow rapidly, converting the surrounding superheated hadronic matter to quark-gluon plasma. Another estimate is obtained by the argument that at these relatively modest beam energies most meson production occurs through the formation and decay of baryon resonances. The most massive observed resonances are in the range of 2 to 2.5 GeV. Putting two of these in close physical proximity leads to a mass of 4 to 5 GeV. Let us assume next that each particle, nucleon and meson, carries away a kinetic energy equal to one half its rest mass. If a particle would have too great a kinetic energy then it might escape from the nucleon-nucleon collision volume long before its neighbors and so would not be counted in the rest mass of the local fluctuation. Taking 4 GeV, dividing by 1.5, and subtracting twice the nucleon mass leaves about 7 pion rest masses. So our most optimistic estimate is that one needs a nucleon-nucleon collision which would have led to 7 pions if it had occurred in free space. One might be less optimistic and require the production of 8 or 10 pions instead.

Conservatively, we concluded by evaluating N_n for $n \geq 7$, that the probability of at least one plasma seed appearing via this mechanism is in the range of 1 to 10% per central gold-gold collision at the highest energy attainable at the AGS.

IV. EXPERIMENTAL SIGNALS

Both homogeneous and inhomogeneous nucleation theory predict, that in rare events quark–gluon plasma can be formed. In both scenarios plasma droplets are formed, but with a rather large uncertainty in both, the parameters and assumptions that go into the theory as well as in the probabilities we obtained. The final answer about the likelihood of rare events of quark–gluon plasma production has to come from experiments.

If a phase transition to quark–gluon plasma occurs in the collision, we can expect a significant additional production of entropy. As a result we should find a strong increase in

the meson production. In general we would expect two distinct two classes of events. One purely hadronic and the rare events, in which a quark–gluon plasma droplet was formed.

These two distinct classes of events might be visible in the charged particle multiplicity distribution which would have the form

$$P_n = (1-q)\, P_n^{\rm had}(N_{\rm had}) + q\, P_n^{\rm qg}(N_{\rm qg})\,. \tag{8}$$

Here q is the probability of finding a central event in which plasma is formed, $P_n^{\rm had}$ is the multiplicity distribution for purely hadronic events with mean $N_{\rm had}$, and $P_n^{\rm qg}$ is the multiplicity distribution for events in which a plasma was formed with mean $N_{\rm qg}$.

To obtain a feeling for the shape and applicability of equation (8) we plot in the Figure to the right different negatively charged particle multiplicity distributions. The mean for purely hadronic events is estimated to be $N_{\rm had} = 145$ and an upper limit of $N_{\rm qg} = 193$ was found for the rare events, under the assumption that all of the matter is converted into plasma [4]. We use Poisson distributions for $P^{\rm had}$ and $P^{\rm qg}$ and plot the negatively charged particle multiplicity distribution defined in eq. (8) for different values of the probability q. A shoulder develops for small q and becomes more pronounced the larger q is.

In summary we propose that the formation of plasma in rare events should have observable consequences for hadron interferometry, deuteron production, and the meson multiplicity distribution. For the multiplicity distribution one would observe a shoulder or second maximum at some multiplicity higher than the most probable one. If there is a phase transition but it is second order or weakly first order then the effect will be much more difficult to see. We eagerly await the results of experiments.

V. REFERENCES

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